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NUMERICAL CALCULATIONS OF TWO-PHASE TURBULENT ROUND JET

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Abstract—The investigation deals with the effect of suspended particles on the dissipation of turbulence energy.

Additional dissipation is hypothesized as caused by the relative velocity between the particles and the fluid, and by structural changes of turbulence.

An extended model for the turbulence energy equation is derived and applied to the case of an axially symmetrical free jet. The governing equations are solved numerically, and the results are compared with experimental data. Reasonably good agreement is obtained.

1. INTRODUCTION

Changes in flow characteristics are known to occur when a second phase is introduced into a fluid. The theoretical prediction of the changes are hindered since complete solutions of complex turbulent flows are lacking and since the interaction between the turbulence of the mainstream and the dispersed phase is not well understood. Finally, there are little experimental data available on this interaction.

The aim of this investigation is to propose a model for two-phase turbulent flows, to solve numerically the flow field in a two-phase (air–solid particles) turbulent jet and to compare the results with experimental data.

2. PREVIOUS STUDIES

2.1 *Models*

Nagarajan & Murgatroyd (1971) presented an analytical model for two-phase (gas–solid particles) turbulent flow in a tube. They considered a steady state problem with low loading, neglecting gravity effects. They assumed linear shear stress and introduced phenomenological coefficients, which enabled them to solve a closed system of equations and to find the relative velocity between the phases and the concentration profile. Later, Nagarajan (1972) extended the model and added gravity and electrostatic forces. In his model he introduced the turbulence energy equation, but neglected all terms except the dissipation and production. Consequently, again a linear shear stress was found and a reduction in turbulence level was observed. This reduction was proportional to the concentration.

Kramer & Depew (1972) proposed a one dimensional model for a fully developed two-phase flow in a vertical tube. They assumed a linear mixing length and expressed the velocity fields in terms of various coefficients which have to be determined experimentally.

Spalding (1971) studied an axially symmetrical turbulent jet of non uniform composition. He solved the conservation equations of momentum and mass, the equation of concentration and the equation for turbulence. In the equation for turbulent energy he introduced a phenomenological coefficient, which made the solution possible.

2.2 *Experimental*

Kada & Hanratty (1960) examined the effect of particulate matter on fluid flow in a tube. Using 0.13–0.25% volumetric concentration they found very little effect on the diffusion. The particles moved independent of one another, with small velocity fluctuations in the radial direction. As they increased the concentration to 0.5%, particles tended to move in groups and they had larger velocities in the radial direction. They also observed large fluctuations in the concentration of the solids. (Obviously, if one wants to avoid particle interactions, concentration lower than 0.5% should be used.)

Vasiliev (1969) surveyed the then available Russian literature. He observed that most research was concentrated on pipe flow and generally a reduction in turbulent energy was noted. He reasoned that the relative velocity between the particles and the main flow cause additional dissipation of turbulent energy. In contrast, Zheltov (1958) found that particles of density higher than that of the fluid, did not follow the fluids average velocity and caused an increase in the turbulence level.

Laats (1966) and Laats & Frishman (1970) studied a dusty, round, turbulent air jet. The dust particles were of 20–60 μm in diameter and they used 100% mass concentration. They found that the normalized velocity distribution was independent of the concentration of the particles, but the jet was narrower and the maximum velocity decayed at a slower rate than a single phase jet. These results were consequently confirmed by Popper *et al.* (1974).

Becker *et al.* (1967) found that the intensity of fluctuation in concentration was about 0.2 for particles of 0.5 μm. Abuaf & Gutfinger (1971) examined large particles of 250–417 μm in an air jet, and found that their motion was almost not effected by the main fluid.

Goldschmidt & Eskinazy (1966) studied two-phase jets, but found no effect of the dispersed phase on the main flow, because they use very low concentration. Hetsroni & Sokolov (1971) studied the interaction between droplets and flow properties of a two-phase turbulent jet. They measured the concentration profiles and the time average and fluctuating velocities of the main stream.

Popper *et al.* (1974) studied the motion of oil droplets of 50 μm, in a round turbulent air jet, using a Laser–Doppler technique. Average velocities of the droplets were measured and it was found that at the jet exit the air velocity was higher than the droplet's velocity while at the fully developed region the droplets' velocity was higher than the air velocity at the same location.

In conclusion, the theoretical models do not account for the interaction between the suspended particles and the turbulence of the main stream, even though such interaction is indicated by experimental data.

The purpose of the present paper is to propose a suitable model for a turbulent, two-phase jet flow. The research was restricted to free jet flow in order to eliminate the very complex phenomena in the immediate vicinity of solid wall, which are not well understood even for single-phase flows. The results were compared with the experimental data.

3. MATHEMATICAL ANALYSIS

3.1 *Modelling turbulence*

One of the main difficulties in solving turbulent flows is to model the so-called Reynolds stresses. There is a voluminous literature on the subject and we will not elaborate on it (c.f. Launder *et al.*, 1972; Wolfshtein *et al.*, 1975).

Most of the models which do not solve a transport equation for the cross product of velocities $\overline{uv}$, use Boussinesq eddy viscosity concept

$$\overline{uv} = -\nu_t \frac{\partial \bar{U}}{\partial y}, \qquad [1]$$

where $\bar{U}$ is the time average velocity in the x direction, and u is its fluctuating component, v is the fluctuating component of the velocity in the y direction and ν_t is the eddy viscosity. For the latter Prandtl proposed

$$\nu_t = l^2 \left| \frac{\partial \bar{U}}{\partial y} \right|, \qquad [2]$$

where l is typical length scale (e.g. the width of the jet).

These models are valid only when the turbulence level is uniquely defined by the mean velocity field. When particles are suspended in the fluid, their influence on the turbulence cannot be predicted without turning to more sophisticated models.

Such models connect the apparent viscosity to the turbulent energy k which is defined as

$$k = \frac{1}{2}[\overline{u^2} + \overline{v^2} + \overline{w^2}]^{1/2}. \quad [3]$$

The eddy viscosity is then given by

$$\nu_t = c_\mu L\sqrt{k}, \quad [4]$$

where c_μ is a constant and L is a length scale, which can be either predicted by a special transport equation or related to the geometry of the flow.

In principle a transport equation for the length scale (or some related property) is preferable to a geometrical perscription of L. However, currently the theoretical basis for such an equation is rather vague (Bradshaw 1975) and an algebraic specification of L seems preferable, at least for jet flows.

3.2 *The governing equations*

The governing differential equations for the mean flows of a turbulent jet carrying suspended particles are now developed. The derivations are in Cartesian coordinates (x_i) but in the final stage the cylindrical form of the equations are presented, as to make it suitable for a round jet. In the derivation the flow is assumed to be stationary and incompressible, and the particles concentration are assumed small. This implies that all terms in the equations containing the mean particle concentration are small. It is also assumed that the particles follow the mean flow but not the turbulent fluctuation. Therefore, the mean particles velocity is nearly equal to the mean velocity of the continuous phase. The validity of this assumption is discussed in the appendix. The flow is assumed to be fully turbulent, and therefore the viscous forces are neglected.

3.2.1 *The mean flow equations.* For a steady incompressible two-phase flow the governing equations are:

Conservation of the continuous phase:

$$\frac{\partial}{\partial t}(1-\phi) + \frac{\partial}{\partial x_j}[(1-\phi)U_j] = 0. \quad [5]$$

Conservation of the particulate phase:

$$\frac{\partial}{\partial t}(\phi) + \frac{\partial}{\partial x_j}(\phi U_{p,j}) = 0. \quad [6]$$

Conservation of momentum in the continuous phase:

$$\frac{\partial}{\partial t}[(1-\phi)U_i] + \frac{\partial}{\partial x_j}[(1-\phi)U_iU_j] = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{\partial^2 U_i}{\partial x_j^2} + F_i \quad [7]$$

where ϕ is the volume fraction of the suspended phase and F is the force applied by the particles on the continuous phase. For a small relative velocity between the particles and the continuous phase, this force is given by the Stokes law

$$F_i = \frac{18\mu}{d^2}\phi(U_{p,i} - U_i) \quad [8]$$

where the subscript "p" denotes the particles.

Equations [5–8] may be reorganized by separating all the quantities to a fluctuating and a time average component in the usual way

$$\phi = \bar{\phi} + \varphi,$$
$$P = \bar{P} + p, \qquad [9]$$
$$U_i = \bar{U}_i + u_i.$$

The equations are simplified by assuming the following:

(i) the flow is stationary;
(ii) concentration of particles is small;
(iii) the difference between the mean velocity and mean particle velocity is small.

The resulting mean flow equations are:

$$\frac{\partial \bar{U}_j}{\partial x_j} = 0, \qquad [10]$$

$$\frac{\partial}{\partial x_j}(\bar{\phi}\bar{U}_j) = -\frac{\partial}{\partial x_j}\overline{\varphi u_{p,j}}, \qquad [11]$$

$$\frac{\partial \bar{U}_i}{\partial t} + \bar{U}_j\frac{\partial \bar{U}_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \bar{P}}{\partial x_i} + \nu\frac{\partial^2 \bar{U}_i}{\partial x_j^2} + \frac{\partial}{\partial x_j}(-\overline{u_i u_j}). \qquad [12]$$

For a round jet, using the boundary layer approximation, these equations reduce to:

$$\frac{\partial \bar{U}}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}(\bar{V}r) = 0, \qquad [13]$$

$$\bar{U}\frac{\partial \bar{\phi}}{\partial x} + \bar{V}\frac{\partial \bar{\phi}}{\partial y} = \frac{1}{y}\frac{\partial}{\partial y}[y(-\overline{v_p\varphi})], \qquad [14]$$

$$\bar{U}\frac{\partial \bar{U}}{\partial x} + \bar{V}\frac{\partial \bar{U}}{\partial y} = \frac{1}{y}\frac{\partial}{\partial y}[y(-\overline{uv})]. \qquad [15]$$

The terms on the R.H.S. of the above equations may be approximated in the usual way

$$-\overline{uv} = \nu_t\frac{\partial \bar{U}}{\partial y}, \qquad [16]$$

$$-\overline{v_p\varphi} = \frac{\nu_t}{\sigma_\phi}\frac{\partial \bar{\phi}}{\partial y}. \qquad [17]$$

For calculating the eddy viscosity a turbulence energy model is chosen.

3.2.2 *The turbulence energy equation.* The turbulence energy equation is generated from the fluctuating velocity equation. The equation thus obtained is rather complex, because it contains some new correlations between velocity and particles concentration. However, for small concentrations the equation is simplified, and contains only three terms more than the equation for single-phase turbulence energy:

$$\bar{U}\frac{\partial k}{\partial x}+\bar{V}\frac{\partial k}{\partial y}= \qquad \textit{Convection}$$

$$=\frac{1}{y}\frac{\partial}{\partial y}\left(y\frac{\nu_{\text{eff}}}{\sigma_k}\frac{\partial k}{\partial y}\right) \qquad \textit{Diffusion}$$

$$+\nu_t\left(\frac{\partial \bar{U}}{\partial y}\right)^2 \qquad \textit{Production}$$

$$-c_D\frac{k^{3/2}}{L} \qquad \textit{Dissipation}$$

$$+\frac{18\mu}{d^2}[(\bar{U}_{p,i}-\bar{U}_i)\overline{\varphi u_i}+\overline{(u_{P,i}-u_i)u_i}\,\bar{\phi}+\overline{\varphi(u_{P,i}-u_i)u_i}]. \qquad \textit{Added dissipation} \quad [18]$$

The last three terms of this equation are called "added dissipation" and require some elaboration.

The first added term is porportional to the relative mean velocity between the particles and the fluid. In the present work this quantity is assumed to be small, and therefore this term is neglected.

The second added term is proportional to $\overline{(u_{p,i}-u_i)u_i}$. This term may be assumed to be bounded by

$$2k \geqslant -\overline{(u_{p,i}-u_i)u_i} \geqslant 0 \qquad [19]$$

where the two bounds represent stationary particles (relative to the turbulent fluctuations) for the left bound, and particles which completely follow the turbulent velocity fluctuations for the right boundary. The actual value of this term is dependent on the ratio t_p/t_t, where t_p is the time required for the particle to adjust to velocity fluctuations, while t_t is the typical time of the turbulent eddies. Mathematically, this is represented by

$$-\overline{(u_P-u)u}=2k(1-e^{-B(t_p/t_t)}) \qquad [20]$$

where B is an empirical constant.

The typical times, t_p and t_t, may be estimated by the following formulae:

$$t_p=\frac{\rho_P d^2}{\mu}, \qquad [21]$$

$$t_t=\left(\frac{\mu}{\rho\epsilon}\right)^{1/2}=\left(\frac{\mu L}{\rho k^{3/2}}\right)^{1/2} \qquad [22]$$

and

$$\frac{t_p}{t_t}=\frac{\rho^{3/2}k^{3/4}d^{3/2}}{\mu^{3/2}}\left(\frac{\rho_P}{\rho}\frac{d}{L}\right)^{1/2}. \qquad [23]$$

The third added term is a triple correlation of velocities and concentration. No attempt has been made in the present study to model this term separately.

3.2.3 *The turbulent length scale.* A transport equation can be written also for a turbulent length scale. Bradshaw (1975) pointed out that even in a single-phase flow the scale equation is somewhat speculative. On the other hand, previous experience suggests that the turbulent length scale is almost uniform in free shear layers. It was therefore decided to use the common description for the length scale:

$$L = \lambda(y_{0.1} - y_{0.9}) \quad [24]$$

where $y_{0.1}$ and $y_{0.9}$ denote the value of the cross stream coordinate y where the velocity reaches 10% and 90%, respectively, of the maximum velocity at this distance x from the outlet nozzle. The constant λ will be so determined as to obtain good agreement with experimental data.

3.3 *Assessment of the mathematical problem*

The problem is solved by solving the system of four equations [13–15], and [18] for the variables $\bar{U}$, $\bar{V}$, $\bar{\phi}$, and k. This is a mixed system, which is nearly parabolic. The corresponding boundary conditions are on the axis of symmetry ($y = 0$)

$$\bar{V} = 0,$$

$$\frac{\partial \bar{U}}{\partial y} = \frac{\partial \bar{\phi}}{\partial y} = \frac{\partial k}{\partial y} = 0. \quad [25]$$

At the edge of the jet ($y = 5$)

$$\bar{U} = \bar{\phi} = k = 0. \quad [26]$$

The equations require also an initial condition, which is a prescription of $\bar{U}$, $\bar{V}$, $\bar{\phi}$, k at some distance x.

The following auxiliary relations should be used to complete the sustem of equations:

The flux relations [16], [17].

The turbulent viscosity definition [4].

The length scale definition [24].

The following numerical constants are used as well c_μ, λ, c_D, B, which are discussed below.

3.4 *Reduction of the equations*

It may be easily seen that [14–16] are very similar to one another and may be brought to the common form:

$$\bar{U}\frac{\partial f}{\partial x} + \bar{V}\frac{\partial f}{\partial y} = \frac{1}{y}\frac{\partial}{\partial y}(yJ_f) + d_f \quad [27]$$

where f stands for $\bar{U}$, $\bar{\phi}$, or k, and J_f is the f-flux.

These equations may be simplified by application of the von-Mises transformation, from coordinates (x, y) to coordinates (x, ψ), where $\psi = \int \bar{U} y \, dy$. The resulting equation is of the form

$$\frac{\partial f}{\partial x} = \frac{\partial (yJ_f)}{\partial \psi} + \frac{d_f}{\bar{U}}, \quad [28]$$

Moreover, the continuity equation [13] is automatically satisfied. Patankar & Spalding (1970) have further developed this transformation by introducing a new cross-stream variable, ω, defined by

$$\omega = \frac{\psi}{\psi_E}, \quad [29]$$

where ψ_E is the value of the stream function at the edge of the shear layer. Obviously, ω varies between zero and unity.

In the present study the Patankar & Spalding transformation and method of solution were used.

4. RESULTS AND DISCUSSION

All the computer runs were compared with the experimental data of Hetsroni & Sokolov (1971). In order to enable a reliable comparison of the two-phase results, the empirical constants c_μ, c_D, and λ were adjusted to give good agreement with the single-phase data of these authors. The values chosen are

$$c_\mu = 0.078,$$

$$c_D = 1,$$

$$\lambda = 0.625.$$

The constants were used to compute the two-phase jet reported by Hetsroni & Sokolov (1971). In this case oil droplets having a uniform diameter of 13 μm were suspended in an air stream. The Weber number for this flow was smaller than unity.

The computations showed that the predicted reduction in the turbulence energy level was by far smaller than the measured one, even when the constant B was made very large (thus bringing the added dissipation to its maximum). It became obvious that the particles bring about a change in the structure of the turbulence. In the present model the structure is represented by the length scale. Therefore, structural variations affect the production and dissipation of turbulent energy. As the particles concentration is low in the present case, the structural variations may be assumed to be linear with the particle concentration m. Thus the following corrections were used:

$$\textit{Dissipation} = (1 + A_D\bar{\phi})c_D\frac{k^{3/2}}{L}; \qquad [30]$$

$$\textit{Production} = (1 + A_P\bar{\phi})\nu_t\left(\frac{\partial \bar{U}}{\partial y}\right)^2. \qquad [31]$$

In general A_D and A_P are functions of the droplet size. For the present case the following values were chosen

$$A_D = 2.3 \times 10^6,$$

$$A_P = 0.72 \times 10^6.$$

In figure 1 the distribution of droplets' flux across the jet is depicted. The flux is normalized to the flux at centerline, and is given as the product of the mean velocity $\bar{U}$ and volumetric concentration $\bar{\phi}$. There are four curves in the figure. Curve 1 is the calculated flux distribution for all droplets concentrations. Curve 2 is the average of the experimental data (Hetsroni & Sokolov 1971), while curves 3 are the bounds of these data.

The calculated results are well within the spread of the experimental data. All the distributions are well described by Gaussian curves.

In figure 2 the calculated distribution of time-average velocities are shown at $x/D = 35$ and compared with the experimental data. The agreement between the two sets of curves is very good in the inner part of the jet but deteriorates toward the edge of the jet, where the experimental accuracy is low.

It can be observed that the width of the jet is reduced with the increase of droplet concentration. Generally, such effects are associated with reduction of the turbulence level. Indeed, this trend was suggested in the previous paragraphs.

The narrowing of the jet is further shown in figure 3, where the calculated half width of the jet is plotted.

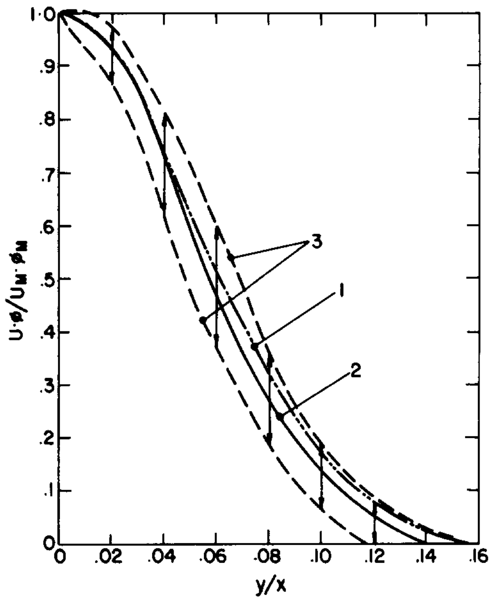

Figure 1. Distribution of droplets flux across the jet. 1, calculated; 2, average of experimental data (Hetsroni & Sokolov 1971); 3, bounds of experimental data.

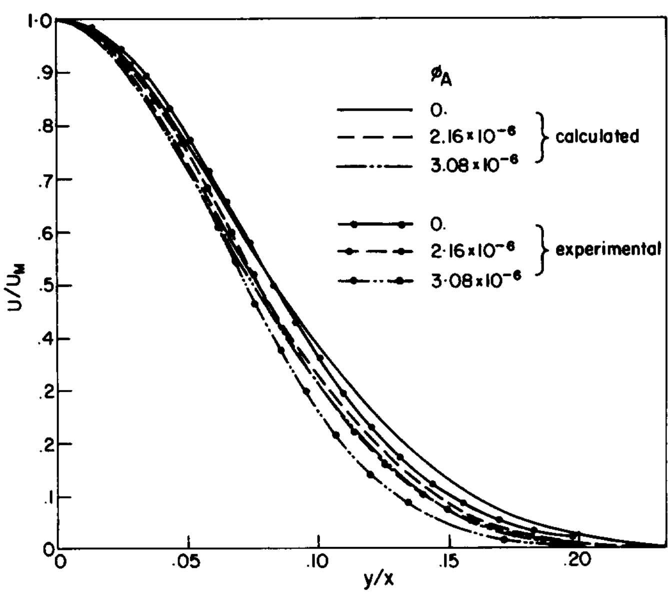

Figure 2. Distribution of time average velocity in a single and two-phase jet at $x/D = 35$.

The half width is still linear with respect to the distance from the nozzle, but the virtual origin of the jet is shifted downstream.

In figure 4 the calculated turbulent energy distributions across the jet are plotted at $x/D = 35$, and compared with the experimental data of Hetsroni & Sokolov (1971). Since hot-wire measurements in two-phase systems is very difficult, the accuracy of the experimental data is relatively low. Therefore, only qualitative agreement between the calculated curves and the data can be expected, and indeed only such agreement was obtained. As suggested above, a significant reduction in the turbulent energy is observed in the two-phase jet. The reduction of energy level is not linearily proportional to the concentration.

The calculated Reynolds stress distribution across the jet at $x/D = 35$ is shown in figure 5. Again, the addition of particles reduces the Reynolds stress.

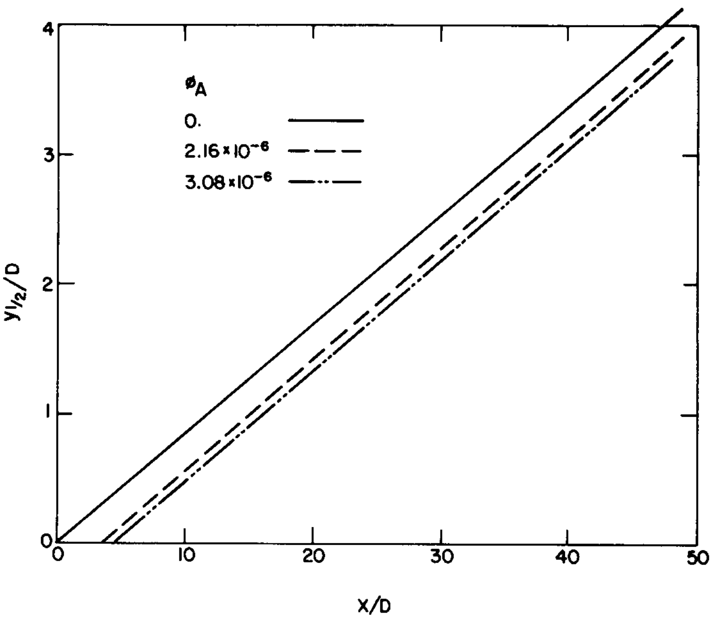

Figure 3. Calculated half width of the jet.

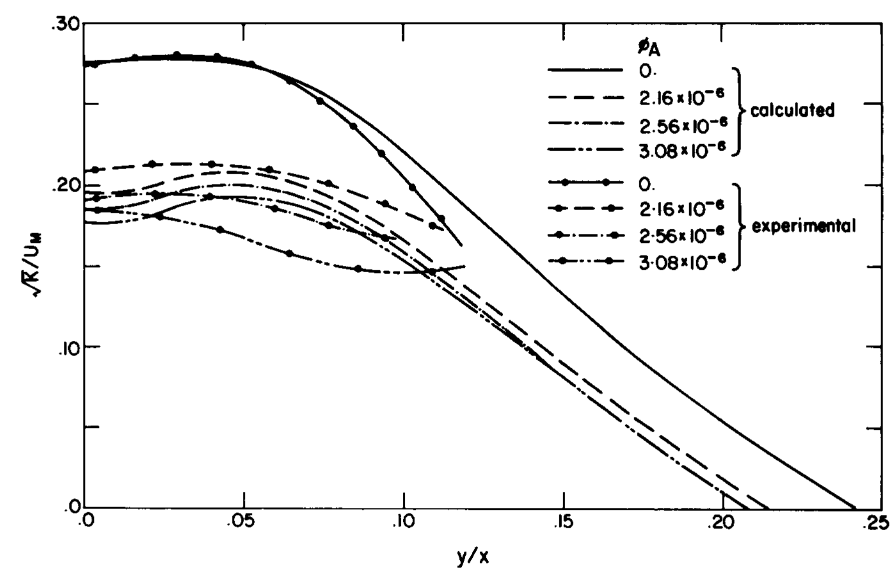

Figure 4. Distribution of turbulent energy across the jet at $x/D = 35$.

5. CONCLUSIONS

The dispersed phase causes a reduction in turbulence energy level of the flow. The simplistic hypothesis that this reduction is due to viscous energy dissipation around the particles is not sufficient to explain this phenomenon.

Consequently, it must be postulated that the particles cause other changes to the structure of the turbulence. We propose to model these changes as described by [30] and [31]. By adding these terms we were able to obtain reasonably good agreement with the experimental results.

Still, the mechanism of the interaction between the dispersed phase and the turbulence is not well understood and additional work, experimental as well as theoretical, is needed.

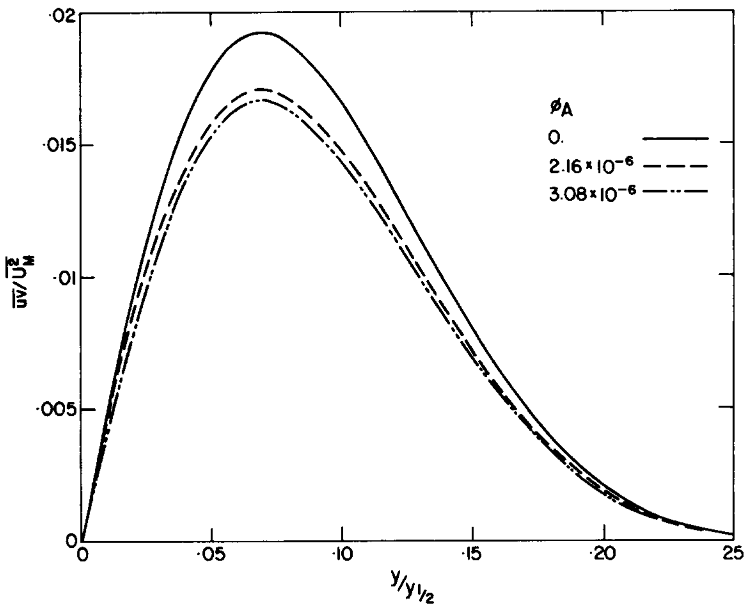

Figure 5. Reynolds stress across the jet at $x/D = 35$.

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APPENDIX

TYPICAL TIMES

In a two-phase turbulent jet several time scales may be defined:

(i) *Mean-flow time* is the time required for a small bulk of fluid to travel one local jet width. This quantity varies across the jet, and its representation is given by

$$t_M = \frac{\delta}{\bar{U}_M} \qquad \text{[A-1]}$$

where δ and $\bar{U}_M$ are the local jet width and maximum velocity.

(ii) *Small-scale-turbulence time* is the time scale of the small dissipative structures in the flow field. It is given by

$$t_t = (\nu/\epsilon)^{1/2} \qquad \text{[A-2]}$$

where ϵ is the dissipation rate.

(iii) *Particles time* is the time scale which is related to the time required for the velocity difference between a spherical particle and the mean flow to vanish when the mean flow velocity is constant. It is given by

$$t_p = \frac{\rho_p d^2}{\mu}$$

where ρ_p is the particle density and μ is the viscosity of the continuous phase.

The ratios of the above time scales define the character of various processes in the flow. Two such ratios are important in the present case:

$$N_{RV} = \frac{t_p}{t_M} = \frac{\rho \bar{U}_M \delta}{\mu} \frac{\rho_p}{\rho} \frac{\delta^2}{d^2}, \qquad \text{[A-4]}$$

$$N_{Tp} = \frac{t_p}{t_t} = \left(\frac{\rho k^{1/2} \delta}{\mu}\right)^{3/2} \frac{\rho_p}{\rho} \frac{d^2}{\delta^2}, \qquad \text{[A-5]}$$

where the relation $\epsilon \propto k^{3/2}/\delta$ was used. Clearly, N_{RV} is proportional to the relative velocity between the particle and the fluid and presents an indicator to the importance of the relative velocity.

In a similar way, when N_{Tp} is large, the particles do not follow the small eddies, while a small N_{Tp} indicates that the particles closely follow the turbulent fluctuations.

In the present computer runs the typical values are as follows:

$$\delta \simeq 0.15X, \qquad \bar{U}_M \simeq 8U_0\frac{b_0}{X}, \qquad d = 1.3 \times 10^{-6}\,\text{m},$$

$$\rho = 1.4\,\text{kg/m}^3, \qquad \rho_p = 1000\,\text{kg/m}^3, \qquad \mu = 2 \times 10^{-5}\,\text{kg/m}\cdot\text{sec}, \qquad k \simeq 0.1\,\bar{U}_M^2$$

at the outlet nozzle

$$b_0 = 0.025\ \text{m}, \qquad U_0 = 50\ \text{m/s}.$$

Therefore,

$$N_{RV} = 5.94 \times 10^9 \left(\frac{X}{b_0}\right)^2$$

which is very large. Therefore, the particles follow the mean flow.

$$N_{Tp} = 519 \left(\frac{X}{b_0}\right)^{-2}$$

which is of order unity for $X/b_0 = 20 \div 30$. Therefore the particles do not follow the small turbulent fluctuations.